Roadmap

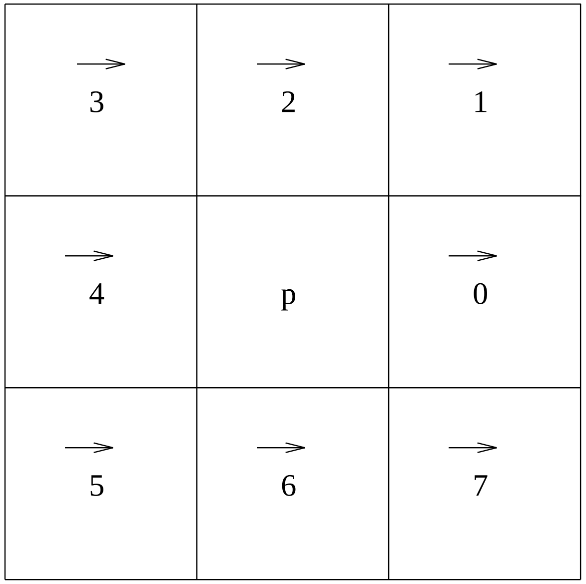
Landmark Recognition

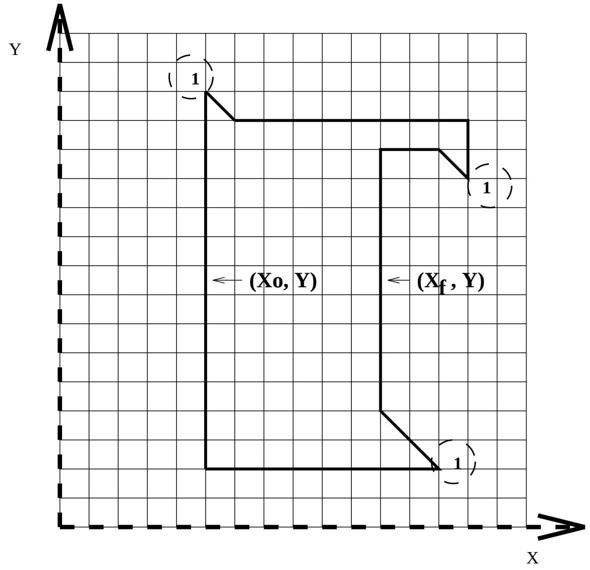
Placing Landmarks

Localization and Navigation

Occluding Objects

Object's Boundary





(a) Freeman's chain-code

(b) Segment based computations

Figure 1: Computing along the object's boundary

Computing Moments

$$
\begin{tikzcd}
\int y^4 \int f & & & & \\
\int y^3 \int f \arrow[u, "\times"] \arrow[r, dotted] & \int y^3 \int x f & & & \\
\int y^2 \int f \arrow[u, "\times"] \arrow[r, dotted] & \int y^2 \int x f \arrow[u, "\times"] \arrow[r] & \int y^2 \int x^2 f = \int x^2 \int y^2 f & & \\
\int y \int f \arrow[u, "\times"] \arrow[r, dotted] & \int y \int x f = \int x \int y f \arrow[u, "\times"] \arrow[r, "\times"] & \int x^2 \int y f \arrow[u, dotted] \arrow[r, "\times"] & \int x^3 \int y f & \\
\int \int f \arrow[u, "\times"] \arrow[r, "\times"] & \int x \int f \arrow[u, dotted] \arrow[r, "\times"] & \int x^2 \int f \arrow[u, dotted] \arrow[r, "\times"] & \int x^3 \int f \arrow[u, dotted] \arrow[r, "\times"] & \int x^4 \int f
\end{tikzcd}
$$

Figure 2: A possible arrangement of terms for computation of moments.

Projection on the Axes

	0	1	2	3	4	5	6	7
0	$+$	$+$	$\neg\exists$	$\neg\exists$	u	u	$+$	$+$
1	$+$	$+$	$\emptyset$	$\emptyset$	$\neg\exists$	u	$+$	$+$
2	$+$	$+$	$\emptyset$	$\emptyset$	$\neg\exists$	$\neg\exists$	$+$	$+$
3	u	u	$-$	$-$	$-$	$-$	$\neg\exists$	u
4	u	u	$-$	$-$	$-$	$-$	$\neg\exists$	$\neg\exists$
5	$\neg\exists$	u	$-$	$-$	$-$	$-$	$\emptyset$	$\emptyset$
6	$\neg\exists$	$\neg\exists$	$-$	$-$	$-$	$-$	$\emptyset$	$\emptyset$
7	$+$	$+$	$\neg\exists$	u	u	u	$+$	$+$

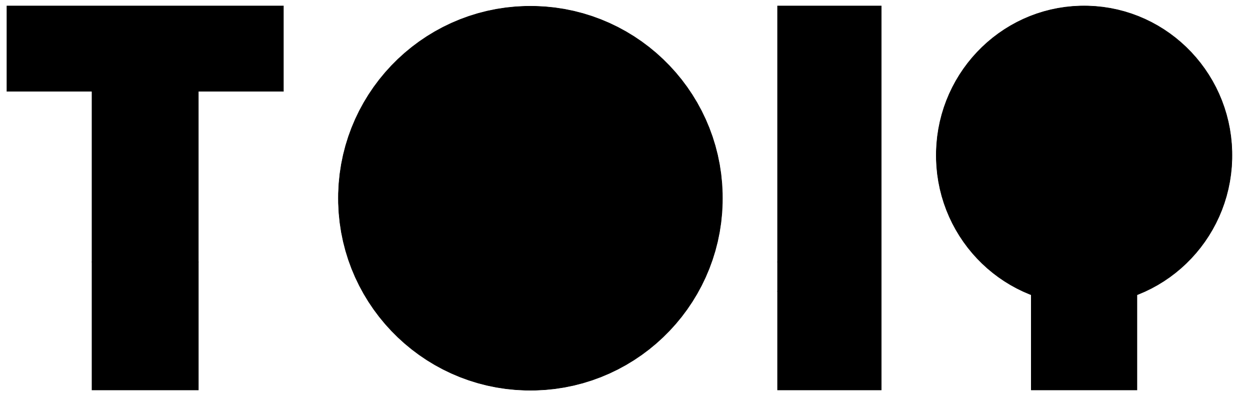
(a) Group M

	0	1	2	3	4	5	6	7
0	$\emptyset$	$\emptyset$	$\neg\exists$	$\neg\exists$	$+$	$+$	$+$	$+$
1	$-$	$-$	$-$	$-$	$\neg\exists$	u	u	u
2	$-$	$-$	$-$	$-$	$\neg\exists$	$\neg\exists$	u	u
3	$-$	$-$	$-$	$-$	$\emptyset$	$\emptyset$	$\neg\exists$	u
4	$-$	$-$	$-$	$-$	$\emptyset$	$\emptyset$	$\neg\exists$	$\neg\exists$
5	$\neg\exists$	u	u	u	$+$	$+$	$+$	$+$
6	$\neg\exists$	$\neg\exists$	u	u	$+$	$+$	$+$	$+$
7	$\emptyset$	$\emptyset$	$\neg\exists$	u	$+$	$+$	$+$	$+$

(b) Group N

Figure 3: Contribution for the cases M (a) and N (b) given two chain-code boundary directions $\mathbf{a}_{i-1}$ (previous, the columns) and $\mathbf{a}_i$ (current, the rows). The symbols $\emptyset, \neg\exists$ mean that no computation takes place, and that the configuration is invalid respectively.

Figures



(a) T (b) O (c) I (d) Q

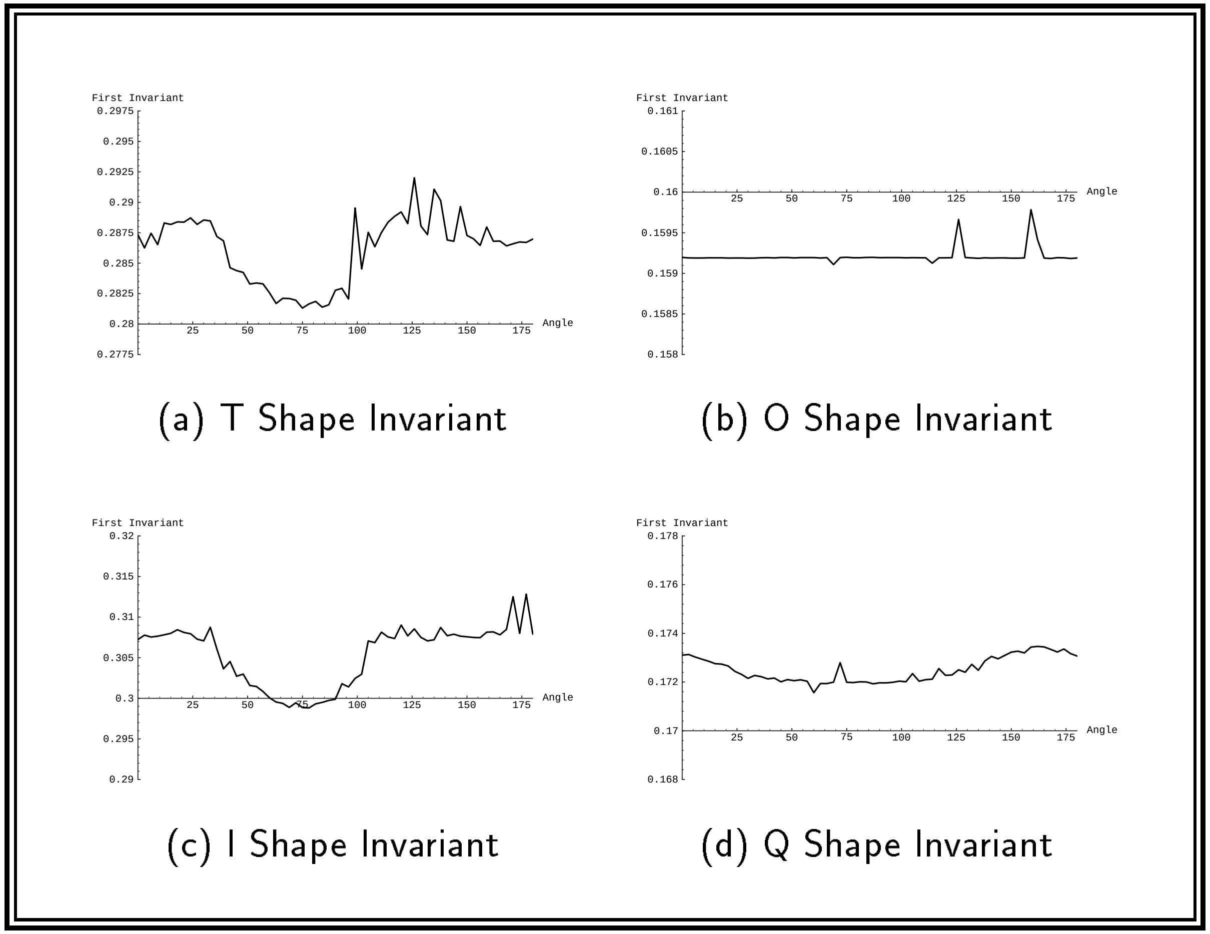
Figure 4: Figures used during the experimentation.

Factoring Terms

m_{pq}	M_+	M_-	M_u	N_+	N_-	N_u
m_{00}	y	$(1-y)$	1	x	$(1-x)$	1
m_{10}	xy	$x(1-y)$	x			
m_{01}				yx	$y(1-x)$	y
m_{20}	x^2y	$x^2(1-y)$	x^2			
$2m_{11}$	$xy(y+1)$	$x(1-y)y$	$2xy$	$yx(x+1)$	$y(1-x)x$	$2yx$
m_{02}				y^2x	$y^2(1-x)$	y^2
m_{30}	x^3y	$x^3(1-y)$	x^3			
$2m_{21}$	$x^2y(y+1)$	$x^2(1-y)y$	$2x^2y$			
$2m_{12}$				$y^2x(x+1)$	$y^2(1-x)x$	$2y^2x$
m_{03}				y^3x	$y^3(1-x)$	y^3

Table 1: Kernel for the computation of moments

First Invariant



(a) T Shape Invariant

(b) O Shape Invariant

(c) I Shape Invariant

(d) Q Shape Invariant

Figure 5: First order moment invariant when the figure rotates 180 degrees around the cameras optical axis

Comparison

Method	Multiplications	Additions
Delta [ZVZMvK87]	$25n$	$n^2 + 21n$
Integral [DBN92]	$8n$	$22n$
Line-Segment [Li93]	$6n + 64$	$n^2 + 17n + 64$
Green's [LS91]	0	$128n$
Interline	$9n + 3$	$12n$

Table 2: Comparison of different methods for computing $m_{pq}, p + q \leq 3$. Data took as published.

Visibility Region

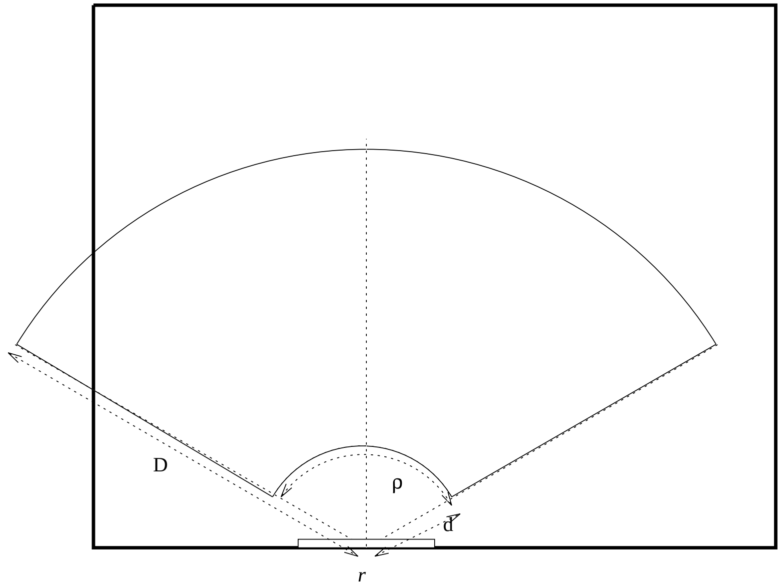


Figure 6: Landmark Positioning Problem. A wall landmark at r can be observed from a distance x, $d < x < D$. The field of view is spanned over ρ.

Simulated Annealing Schedule

Initial temperature. $t \leftarrow 1.0$

Factor. 0.95

Temperature Steps. 100

Combinations at any temperature. 100 $\times$ number of landmarks.

Succesful changes. 10 $\times$ number of landmarks.

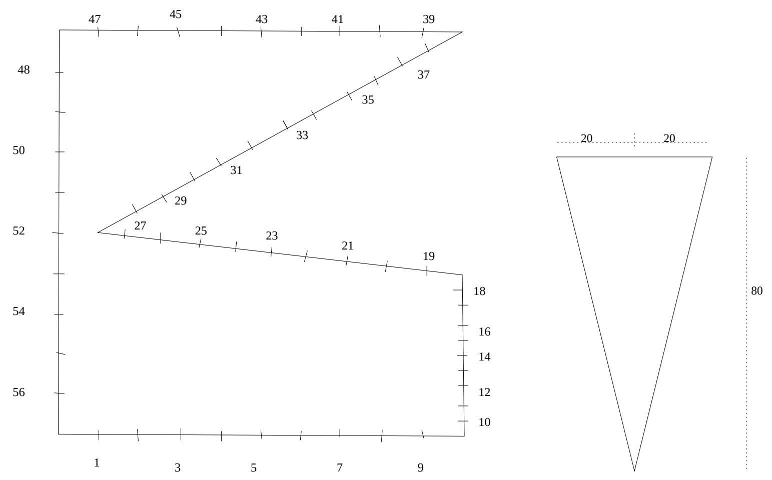
Event. (Area$_{current}$ - Area$_{new}$)/(100 t)

Objective Function.

$$A(\bigcup U_i) = \sum_{i=1}^{m} A(U_i) - \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} A(\bigcap(U_i, U_j)) + O(A(\bigcap_3)) \qquad (1)$$

Oracle. $y < \exp -\lambda/t$

Workspace-Observability Region

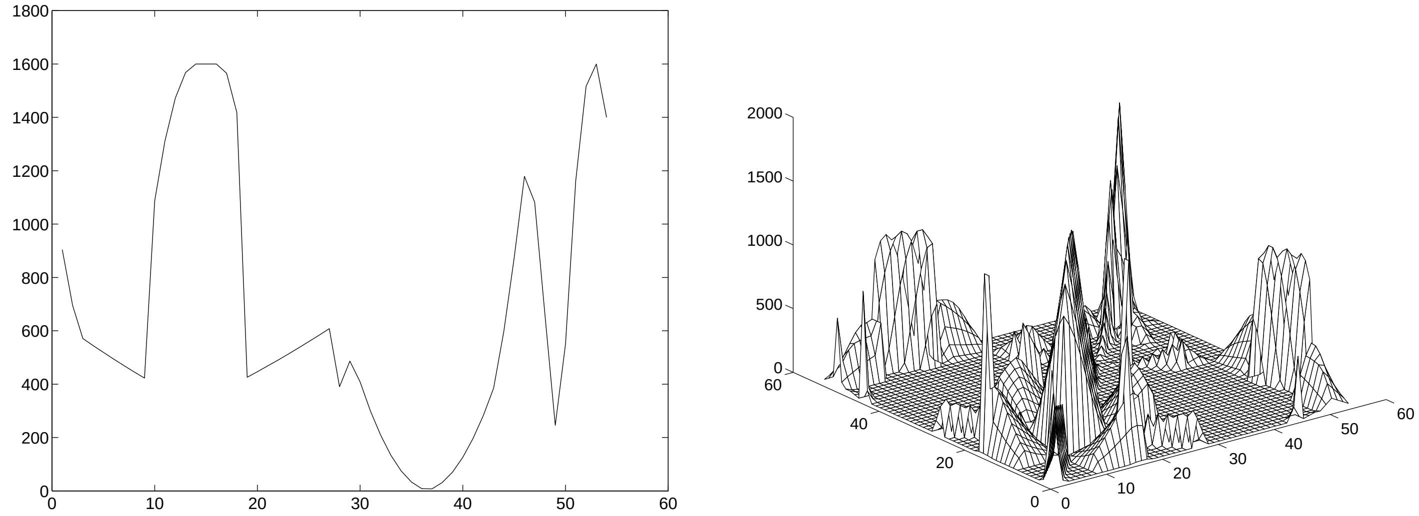


(a) Workspace

(b) Visibility Region

Figure 7: Polygons used in the example

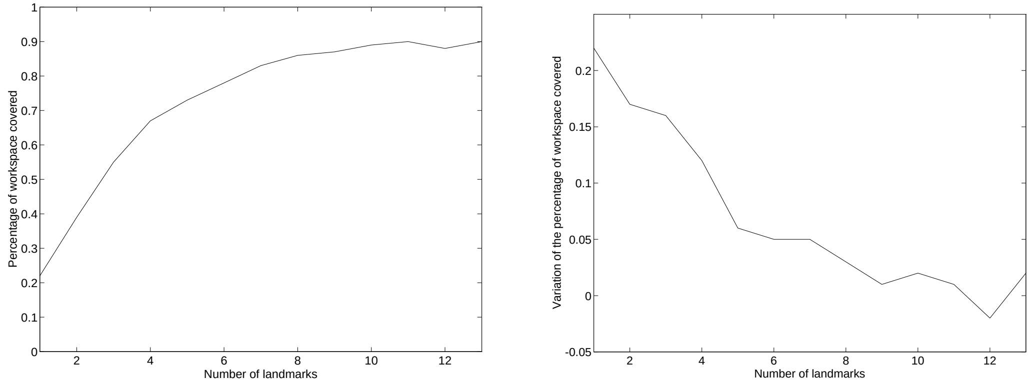
Visibility



(a) Single Landmark (b) Pairs of landmarks

Figure 8: Portion of the workspace covered by visibility regions

Evaluation Parameters



(a) Occupancy Level (b) Occupancy Grow

Figure 9: The horizontal axis shows the number os landmarks

Landmark Position

1	15												
2	18	54											
3	18	46	54										
4	1	18	46	54									
5	1	18	44	46	54								
6	1	18	19	44	46	54							
7	18	19	27	45	46	48	54						
8	18	19	27	29	45	46	48	54					
9	9	13	20	27	29	45	46	48	51				
10	1	9	18	19	27	28	44	46	48	54			
11	1	9	18	19	27	28	34	44	46	48	54		
12	8	9	10	15	27	29	31	33	45	46	48	50	
13	9	10	14	27	29	33	34	36	45	46	48	49	51

Table 3: Number of landmarks and position in the workspace

Landmark

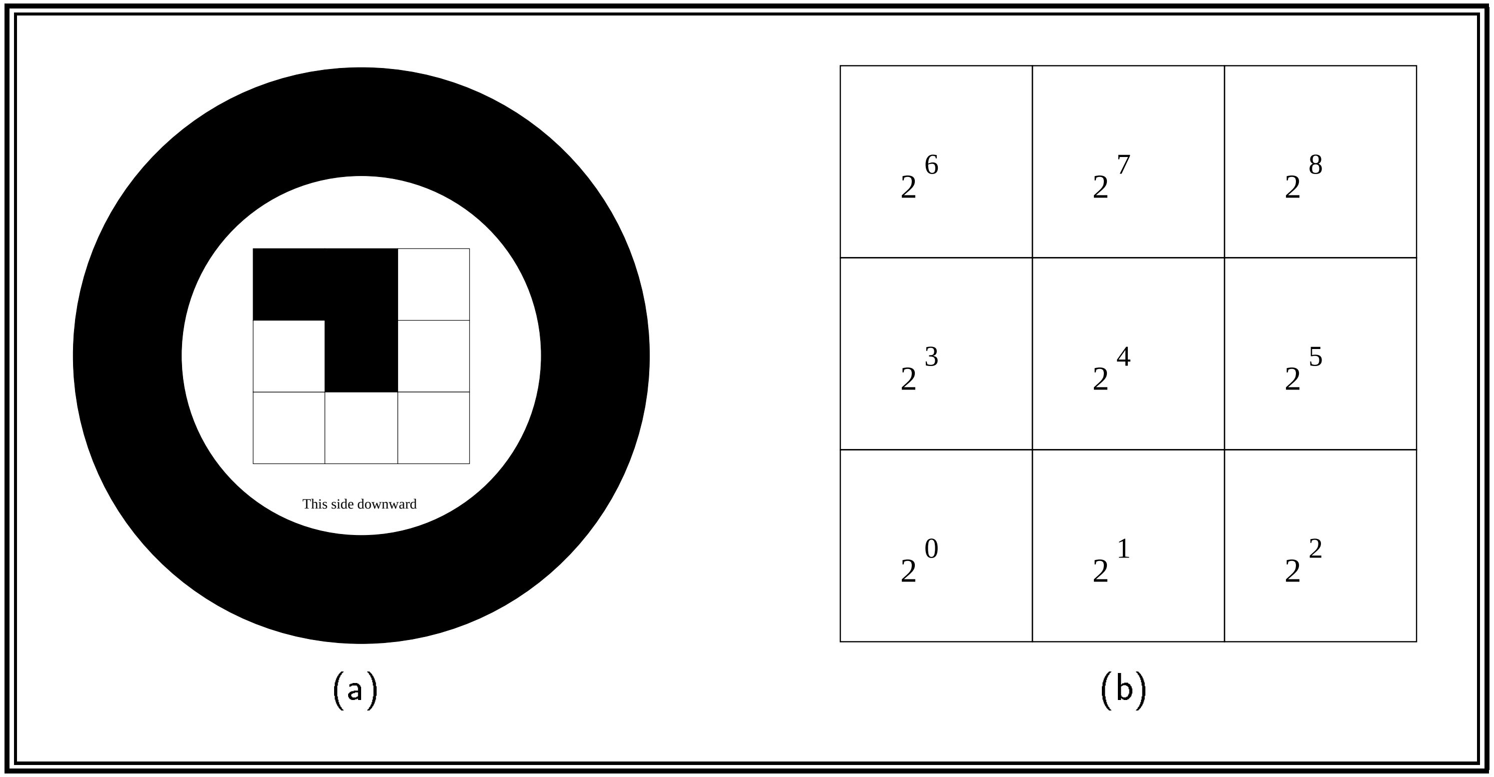


Figure 10: A landmark design (a). (b) shows the relative value of each particular grid's cell. Black and white cells have a value one and zero respectively. The landmark in (a) has an ID number $1 \times 2^4 + 1 \times 2^6 + 1 \times 2^7 = 208$

Localization

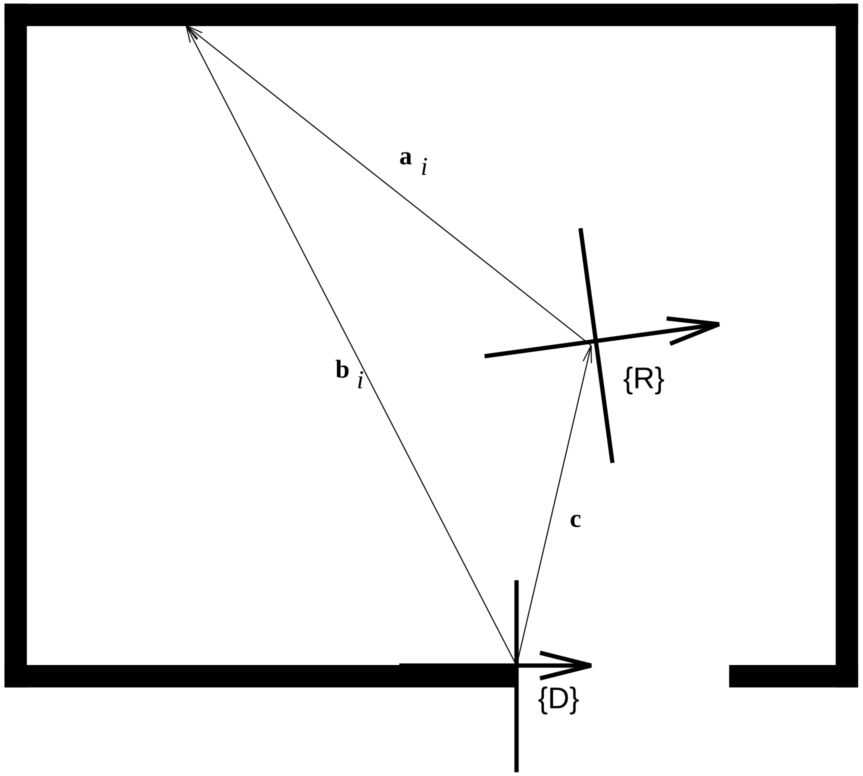


Figure 11: Robot localization as a function of the observations $\mathbf{a}_i^D$ and *apriori* acknowledge $\mathbf{b}_i^D$.

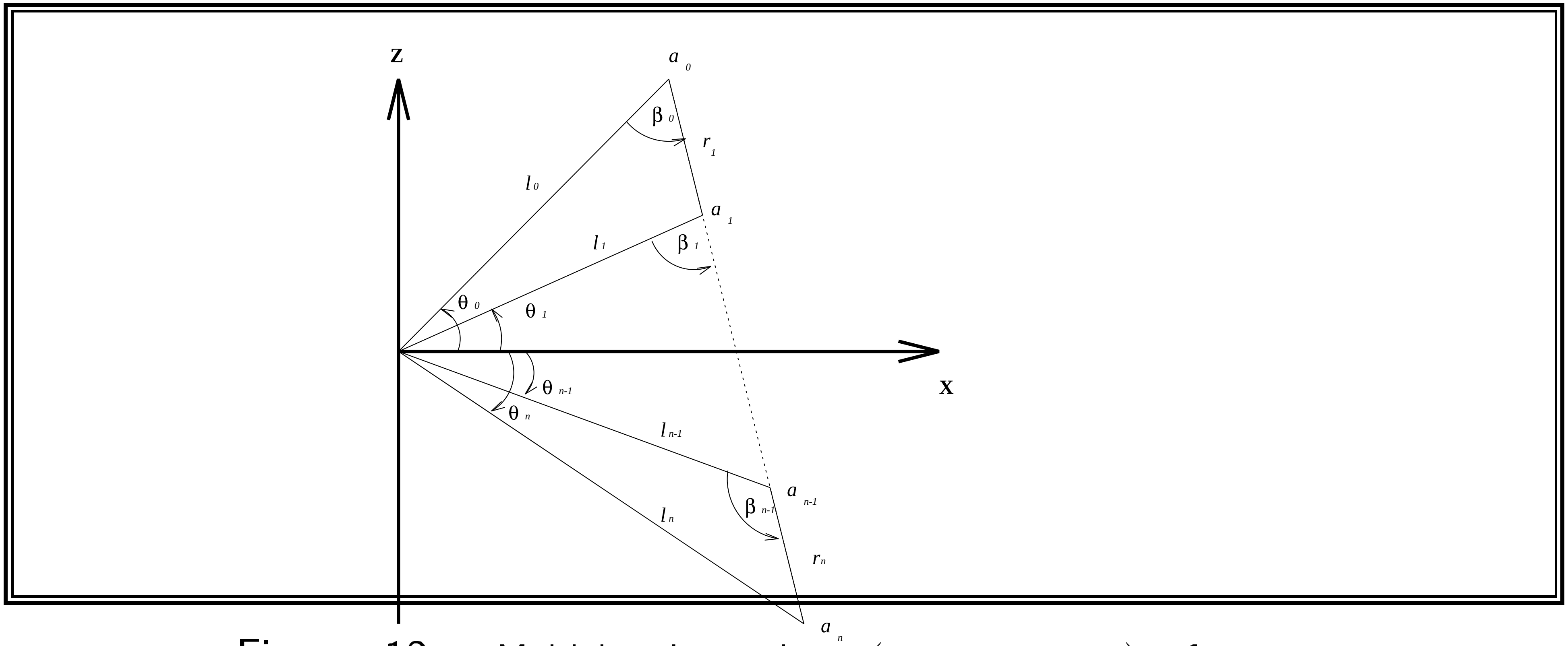


Figure 12: Multiple observations $(a_0, a_1, \ldots, a_n)$ of a landmark centered in the reference system.

Computing Optical Flow

Error Function

$$e(\mathbf{p}, \mathbf{d}_0) = \sum_{\delta_0 \in N(\mathbf{p})} \mid I_i(\delta_0 + p) - I_{i+1}(\delta_0 + p + \mathbf{d}_0) \mid \tag{2}$$

Raw Flow

$$O^*(\mathbf{p}) = \{\mathbf{d}_i | e(\mathbf{p}, \mathbf{d}_i)\} \tag{3}$$

Optical Flow

$$O^(p) = \mathsf{Mode}_V(O^*(p)) \tag{4}$$

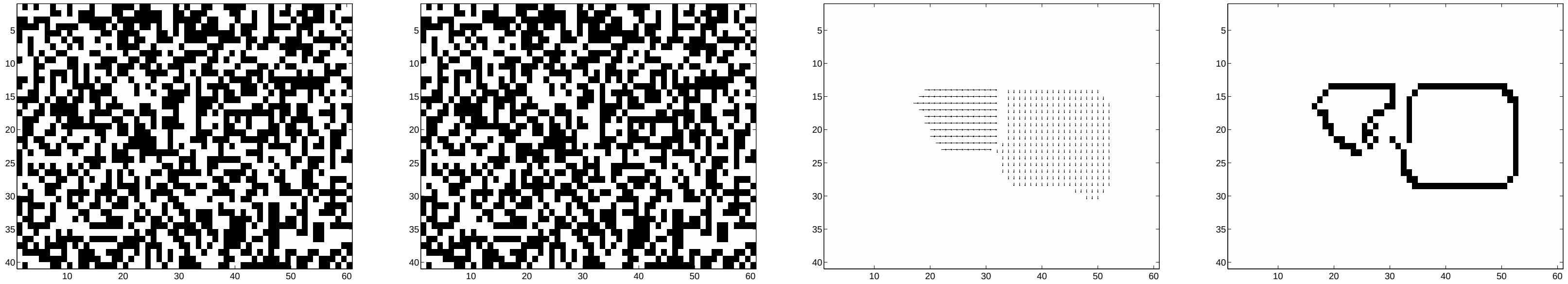
Trajectory Map.

$$T_i^k(p) = \begin{cases} T_{i+1}^k(O_i(p)) + O_i(p) & \text{if } i < k \\ O_i(p) & \text{otherwise} \end{cases} \tag{5}$$

Boundary.

$$B_i(p) = \begin{cases} 1 & |J(T_i^j(p))| \geq \tau \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Synthetic Images



(a) Frame I_0 (b) Frame I_1 (c) Flow O_0 (d) Boundaries

Figure 13: An illustrative example with synthetic images. We compute flow between consecutive frames. The occluding boundaries are edges in the trajectory map.

Summary

Computation of moments

Landmark placing in the workspace

Localization and navigation using landmarks

Detection of occluding objects

References

[DBN92] Mo Dai, Pierre Baylou, and Mohamed Najim. An efficient algorithm for computation of shape moments from run-length codes or chain codes. *Pattern Recognition*, 25(10):1119–1128, 1992.

[Li93] Bing-Cheng Li. A new computation of geometric moments. *Pattern Recognition*, 26(1):109–113, 1993.

[LS91] Bing-Cheng Li and Jun Shen. Fast computation of moment invariants. *Pattern Recognition*, 24(8):807–813, 1991.

[ZVZMvK87] M. F. Zakaria, L. J. Vroomen, P.J. A. Zsombor-Murray, and J.M.H.M. van Kessel. Fast algorithm for the computation of moment invariants. *Pattern Recognition*, 2(6):639–643, 1987.